Final Report:

First-Year Core Calculus

Sciences Calculus
Social Sciences/Business Calculus

June 2002

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BRITISH COLUMBIA COUNCIL ON ADMISSIONS & TRANSFER
SUPPORTING BC's EDUCATION SYSTEM

Final Report:
First-Year Core Calculus

June 2002

This report was prepared by Leo Neufeld, First-Year Core Calculus Project Co-ordinator;
Chair, BC Committee on the Undergraduate Programme in Mathematics (BCcupm).
Assistance from Jennifer Orum in guiding this report to publication is gratefully acknowledged.

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Executive Summary

Initiated in May 2001 by the mathematics/statistics articulation committee (BCcupm), the First-Year Core Calculus project sought to ease the transferability of calculus courses within British Columbia. The objectives were to develop a core curriculum for each of two first-year calculus streams and to recommend their province-wide adoption.

The BCcupm adopted the recommendations of the *First-Year Core Calculus* report unanimously in May 2002.

Recommendations to the BCcupm

1. That the BCcupm accept the report of the Core First-Year Calculus Sub-committee and endorse the Core Sciences and Core Social Sciences/Business curricula as described in it.

2. In making transfer decisions regarding first-year calculus courses, that receiving institutions grant full transfer credit to courses from other BC post-secondary institutions offering calculus courses consistent with the curricula described in the report.

3. When designing or modifying first-year calculus courses, that all BC post-secondary mathematics departments strive to include within their courses the Core Calculus topics as described in the report.

4. That any post-secondary institution sensing that the Core Calculus curricula of the report require a full or partial review raise their concerns at the next regularly scheduled meeting of the BCcupm.

5. In the absence of an earlier full review, that the Core Calculus curricula be subject to a mandatory, full review after five years.

Recommendations to BCCAT

1. That the project team for any future initiative of this kind have a membership that balances representation from each constituency (college, university-college and university) in the post-secondary educational sector.

2. That the universities be strongly represented on such a project team.

3. That a chair and/or facilitator for the project team be clearly identified to act as a liaison with BCCAT and to manage the project.

4. That the final report of this Transfer Innovations Project be sent in hard copy to each post-secondary institution and be posted on the BCCAT web site.

5. That the BCcupm web site contain a separate web page carrying summary information about the core calculus curricula and carry a special link to the BCCAT posting.

Background and Objectives

A. The BCcupm

The British Columbia Committee on the Undergraduate Programme in Mathematics (BCcupm) is the post-secondary mathematics and statistics articulation committee for the province. Its activities are conducted under the auspices of the BC Council on Admissions and Transfer (BCCAT). Articulation committees were established to foster and maintain course/program transfer arrangements to assist students in moving easily from one educational institution to another.

At various times, members of the BCcupm had observed that the regular maintenance of mathematics/statistics courses, particularly at some of the smaller institutions, could be assisted significantly by the existence throughout the province of greater uniformity in course content for similar courses. At its May 2001 meeting in Cranbrook, the BCcupm passed the following motion:

> **That, in order to ease transferability across all first-year calculus streams, we strike a subcommittee to develop a list of core topics for all streams of Calculus I and Calculus II.**

The meeting struck a sub-committee with representation from universities, university-colleges and colleges, and directed the subcommittee to prepare a report for the May 2002 meeting of the BCcupm. The members also recommended that financial support be sought to assist the subcommittee in its activities.

B. Objectives of the Transfer Innovations Project

Support for a *First-Year Core Calculus* initiative was sought and obtained from BCCAT in the form of a Transfer Innovations Project grant.

A primary goal of the project was to accomplish the directive of the BCcupm, namely, to assemble a team to develop a core curriculum for all streams of first-year calculus. The achievement of this objective also required:

- That the working group consider the issues and challenges surrounding its task, and develop a reasonable core curriculum for first-year calculus courses.
- A full report, containing recommendations for a calculus core, to be prepared for presentation to the BCcupm.

The goals of the Transfer Innovations Project were that the Contractor would provide evidence of:

- ✓ The development of core curricula for a sciences stream and a non-science stream of first-year calculus.
- ✓ The completion of a full report of the *First-Year Core Calculus* project containing prospects for the improved transfer of post-secondary students within B.C.
- ✓ The encouragement of system-wide adoption of these core curricula with a prospect toward readily accessible publication.

Activities

In July the Core Calculus Subcommittee held its inaugural meeting to clarify its objectives, to identify important issues and to agree on procedural strategies. In subsequent meetings, the Subcommittee, which had also become a Project Team with a Transfer Innovations Project grant from BCCAT, incrementally refined the draft core calculus curriculum outlines for the Sciences and for the Social Sciences/ Business streams. Besides investing personal expertise in the analysis of existing courses and in the creation of the draft outlines, Team members also consulted regularly with colleagues at their institutions and within the constituency (College, University College, University) they represented.

In all the Team met, with full attendance, on five occasions. With the encouragement and support of BCCAT and of the mathematics/statistics community, and in a spirit of genuine co-operation, these sessions proved to be very productive.

On May 24, 2002, the report of the project was presented to the BCcupm. All but two public BC post-secondary institutions were represented at the meeting and forty-two delegates attended this session.

Following the presentation and with extensive discussion, the BCcupm unanimously adopted individually each of the five recommendations. Besides noting its cordial appreciation for the efforts of the Project Team, the BCcupm also recommended that publication of the report occur in a manner consistent with the Team's recommendations to BCCAT.

> The Core Project arose from a transfer problem faced by sending institutions and their students. The scope of the recommendations presented by the Core Committee to BCcupm to solve the problem went far beyond original expectations in providing guidelines for both sending and receiving institutions.
>
> —David Leeming, University of Victoria

Findings and Outcomes

A. ISSUES AND CHALLENGES

Besides examining the principal issue, that of the difficulties faced by mathematics departments attempting to prepare students for transfer to a number of other post-secondary educational institutions, the Project group also identified related considerations. The group realised that an awareness of these parallel issues and challenges was critical to any hope of success in its primary task.

Some of the considerations explicitly named and in no particular order were:

1. There is an inter-flow of students between science and non-science programs even within post-secondary institutions.
2. Post-secondary institutions are autonomous.
3. Prerequisites for entry to present calculus courses are not consistent throughout the province.
4. In calculus course delivery, there are differences in the sequence in which topics are delivered from institution to institution.
5. Although not unique to the considerations for this activity, the differing policies with respect to the inclusion of technology both in instruction and in evaluation needs to be kept in mind.
6. Can core curricula be so described as to avoid a gradual degeneration of some existing calculus programs? Maintaining standards is vital.

B. PATHS TO AGREEMENT

The successful completion of similar projects may benefit from these observations and experiences:

1. The BCcupm at its annual meeting designated the composition and some of the members of what ultimately became the Transfer Innovations Project team.
2. The strong involvement of Receiving institutions in this process was essential.
3. Rather than beginning anew, the Project Team chose one institution's calculus courses to adopt or enhance in the core curricula development process.
4. Focusing on the benefits to Sending institutions and their students of an approved core curriculum proved very helpful.
5. Following adoption of the core curriculum, one might first expect that any courses falling within the guidelines would, upon application, receive full transfer credit.

> I believe that the mere existence of such a report and its strong endorsement from the BCcupm indicate that all BC institutions consider transferability of calculus courses to be a relevant and important issue in designing/revising the core curriculum.
>
> —Rustum Choksi, Simon Fraser University

C. ANCILLARY OUTCOMES

In addition to successfully accomplishing its primary tasks, the Project Team noted that the initiative had resulted in several supplementary benefits and outcomes.

The Team members from various post-secondary institutions concentrating on the review and development of curricula created a synergistic environment that promoted:

- A clearer, mutual understanding of the dynamics involved in the design and articulation of similar courses at distinct, post-secondary institutions.

- A sharpened awareness of the significant, existing similarities in the aims and practices of mathematics education among post-secondary institutions.

- A heightened sensitivity among Team members to the challenges faced by transferring students and to the advantages to these students of greater uniformity in entry-level courses.

> Even intense negotiations can be enjoyable when everyone involved shares the same overall objective. This Project gave me valuable insight into the issues and concerns of a variety of institutions around the province, and helped strengthen the network of goodwill and collaborative spirit that unites mathematicians here and abroad.
>
> —Philip Loewen, The University of British Columbia

D. THE CORE CURRICULA

1. SCIENCES CALCULUS

First-Year Core Calculus – Sciences Stream

A first year (two-semester) Sciences Calculus course must include all the topics from the Core Topics list. It is expected that coverage of this material would constitute three-quarters of the course(s) with the remaining one-quarter chosen from the Additional Topics list. For breadth, at least four Additional Topics should be included.

Reference Text: Edwards & Penney, *Calculus, Early Transcendentals, Fifth Edition*, Prentice Hall, 1998.

Core Topics (75%)

1. Limits, continuity, intermediate value theorem
2. Differentiation
   - First and second derivatives with geometric and physical interpretations
   - Mean value theorem
   - Derivatives of exp and log functions, exponential growth and decay
   - Derivatives of trigonometric functions and their inverses
   - Differentiation rules (including chain rule, implicit differentiation)
   - Linear approximations and Newton's Method
   - Optimization - local and absolute extrema with applications
3. Taylor polynomials and special Taylor series (sin, cos, exp, 1/(1-x))
4. Curve sketching
5. Integration
   - Definition of the definite integral
   - Areas of plane regions
   - Average value of a function
   - Fundamental Theorem of Calculus
   - Integration techniques: substitution (including trig substitution), parts, tables, partial fractions
   - Applications of integration
6. Numerical Integration (including The Trapezoidal Rule)
7. Improper integrals: evaluation and convergence estimates
8. Differential equations (first-order linear) with applications

Additional Topics (25%)

1. Sequences and series
2. Arc length, volumes, centroids, surface areas
3. Additional differential equations topics
4. Complex numbers
5. Continuous probability density functions
6. Polar coordinates and parametric equations (with calculus applications)
7. Additional numerical methods (eg. Simpson's Rule)
8. Related rates
9. L'Hôpital's Rule

2. SOCIAL SCIENCES/BUSINESS CALCULUS

First-Year Core Calculus – Social Sciences/Business Stream

A first year (two-semester) Social Sciences/Business Calculus course must include all the topics from the Core Topics list. It is expected that coverage of this material would constitute approximately two-thirds of the course(s) with the remaining one-third chosen from the Additional Topics list. For breadth, at least four Additional Topics should be included.

Reference Text: Haeussler and Paul, *Introductory Mathematical Analysis for Business, Economics, and the Life and Social Sciences, Ninth Edition*, Prentice Hall, 1998.

Core Topics (67%)

1. Limits, continuity, intermediate value theorem
2. Differentiation
    - First and second derivatives with geometrical and physical interpretations
    - Applications to economics, business and social sciences
    - Derivatives of exp and log functions, exponential growth and decay with applications
    - Derivatives of trigonometric functions
    - Differentiation rules (including chain rule, implicit differentiation)
    - Linear approximations and Newton's Method
    - Optimization - local and absolute extrema with applications
3. Curve sketching
4. Integration
    - Definition of the definite integral
    - Areas
    - Average value of a function
    - Fundamental Theorem of Calculus
    - Integration techniques: substitution, parts, tables
    - Applications of integration
5. Numerical integration (including The Trapezoidal Rule)
6. Differential equations (first-order linear) with applications

Additional Topics (33%)

1. Introduction to probability and statistics
2. Partial derivatives and Lagrange multipliers
3. Matrix analysis and Gaussian Elimination
4. Sequences and series
5. Arc length, volumes, centroids, surface areas
6. Taylor polynomials and special Taylor series (sin, cos, exp, 1/(1-x))
7. Improper integrals: evaluation and convergence estimates
8. Continuous probability density functions
9. Related rates
10. Derivatives of inverse trigonometric functions
11. Further techniques of integration
12. Additional numerical integration methods

Conclusions and Recommendations

A. RECOMMENDATIONS TO THE BCCUPM

The Project Team prepared the following statement and recommendations for consideration by the BCcupm with respect to the accompanying Core Curricula for Calculus:

The BCcupm affirms the autonomy of post-secondary institutions in BC in their freedom to design calculus courses to meet the needs of their unique constituencies. However, we wish to address the problems encountered by transferring students regarding first-year calculus.

The diversity of calculus courses in 1st year offerings in BC post-secondary institutions has created problems for transferring students and their institutions. For example, while current first-year "business" calculus courses at SFU, UBC and UVic share many core topics, the additional material covered by each institution is irreconcilable within a single two-term calculus sequence.

In order to address this problem, we have created common and additional topics lists for first-year calculus to enable post-secondary institutions to provide these benefits:

a) To offer transferring students academic preparation to ensure greater compatibility with subsequent math courses.
b) To allow primarily sending institutions to design calculus courses that will transfer smoothly.
c) To guide primarily receiving institutions in assessing the transferability of proposed courses.

We are asking the BCcupm to endorse the following recommendations:

1. That the BCcupm accept the report of the Core First-Year Calculus Subcommittee and endorse the Core Sciences and Core Social Sciences/Business curricula as described in it.

2. In making transfer decisions regarding first-year calculus courses, that receiving institutions grant full transfer credit to courses from other BC post-secondary institutions offering calculus courses consistent with the curricula described in the report.

3. When designing or modifying first-year calculus courses, that all BC post-secondary mathematics departments strive to include within their courses the Core Calculus topics as described in the report.

4. That any post-secondary institution sensing that the Core Calculus curricula of the report require a full or partial review raise their concerns at the next regularly scheduled meeting of the BCcupm.

5. In the absence of an earlier full review, that the Core Calculus curricula be subject to a mandatory, full review after five years.

B. TRANSFER INNOVATIONS PROJECT RECOMMENDATIONS

With respect to the Transfer Innovations Project, the Team recommended:

1. That the project team for any future initiative of this kind have a membership that balances representation from each constituency (college, university-college and university) in the post-secondary educational sector.

2. That the universities be strongly represented on such a project team.

3. That a chair and/or facilitator for the project team be clearly identified to act as a liaison with BCCAT and to manage the project.

4. That the final report of this Transfer Innovations Project be sent in hard copy to each post-secondary institution and be posted on the BCCAT web site.

5. That the BCcupm web site contain a separate web page carrying summary information about the core calculus curricula and a special link to the BCCAT posting of the Project Report.

What I have learned from this process is that it is critical for sending and receiving institutions alike to sit together and decide, in reasonably broad terms, the content of common courses. While we do this regularly at our annual meetings, focusing on one course or a short sequence enabled us to produce a set of general guidelines for the articulation of the most common mathematics courses. I would imagine that first-year courses in most subject areas and at most institutions are relatively similar. The process of agreeing on topic lists allows colleges to become clear about the expectations of the universities and allows universities to have confidence in the outcomes. Within the atmosphere of trust and confidence that a collaborative approach engenders, the focus turns away from the subject material and toward the student.

A continuing issue that is difficult to resolve is that of depth or standards. Short of having common final exams such as those given grade 12 students, I don't know how anyone could ever be certain that the standard at one institution is the same as that at another. The present system of agreements based upon the exchange of exams, personal contact between university and college faculty and institutional research of student outcomes post-transfer may appear cumbersome and open to abuse. It is, however, a system that allows for a reasonable amount of autonomy and flexibility and is in keeping with the nature of our enterprise, specifically, education.

I would encourage the BCcupm to continue to review the course content for other first and second year courses. I would furthermore encourage other disciplines to examine their first and second year offerings in a similar manner.

—Bruce Kadonoff, Coquitlam College

The BCcupm's unanimous approval of the first-year core calculus curriculum proposal demonstrates a genuine commitment to compromise and cooperation among all institutions. I believe this has resulted in a significant step towards the goal of providing students a seamless transition from sending to receiving institution.

—Casey McConill, Kwantlen University College

Project Team

The members of the Project Team with their affiliations were:

Rustum Choksi (Simon Fraser University)
Bruce Kadonoff (Coquitlam College)
David Leeming (University of Victoria)
Philip Loewen (The University of British Columbia)
Casey McConill (Kwantlen University College)
Leo Neufeld (Chair, BCcupm, and Project Contractor)

Appendix:

REPORT TO THE BCCUPM

First-Year Core Calculus

May 2002

Core Calculus Subcommittee:

Bruce Kadonoff, Chair
Rustum Choksi
David Leeming
Philip Loewen
Casey McConill
Leo Neufeld

Acknowledgments

We sincerely thank Kwantlen University College and Simon Fraser University for their generous provision of ideal meeting spaces for the Core Calculus Subcommittee. Conveniently located in supportive environments, these accommodations assisted significantly in promoting productivity and success at our sessions.

Without the consent of the mathematics/statistics departments represented by Core Calculus Subcommittee members, this initiative could not have begun. We are grateful to each department for its understanding and support in providing a strong, willing participant for the work of the committee.

Not least, we acknowledge the enthusiastic support and assistance given to this undertaking by the BC Council on Admissions & Transfer. BCCAT and, particularly, the staff members in the BCCAT Office have cheerfully and competently offered expert advice and assistance throughout the process. We cordially thank them for their vision and guidance.

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Executive Summary

This initiative, whose goal is to ease the transferability of first-year calculus courses within BC, was formally launched by the BC Committee on the Undergraduate Programme in Mathematics (BCcupm) at its meeting in May 2001. An *ad hoc* committee of the BCcupm was formed to "develop a list of core topics" for first-year calculus streams and was asked to report to its May 2002 meeting.

Activities

The BC Council on Admissions & Transfer (BCCAT) approved a Transfer Innovations Project application to support financially the Subcommittee's activities. This grant enabled the Subcommittee to meet on five occasions gain a clear, mutual understanding of the committee's task, to develop efficiently the core calculus curricula proposal and to agree on the recommendations for the BCcupm.

Findings

- ✓ It was found that, in the first-year calculus courses examined, the number of topics common to all was proportionately quite large.
- ✓ When topic placement in first-year calculus courses was compared by semester, considerably less agreement in topic order was observed.
- ✓ In deciding on a set of topics for inclusion in a first-year calculus core, the Subcommittee learned that an "enabling transfer" approach, as opposed to a prescriptive approach, was essential to any expectation of general agreement.
- ✓ Although a core of topics for all first-year calculus courses exists, a marked difference in emphasis was noted between the Sciences Calculus and the Social Sciences/Business Calculus streams.

The Core Calculus Curricula

The Subcommittee developed a Core Curriculum for first-year Sciences Calculus (page A-9) and first-year Social Sciences/Business Calculus (page A-11). To accommodate differing emphases locally, without compromising existing standards, these curricula have been presented in two parts, Core Topics and Additional Topics, whose relative weightings have been specified.

Recommendations

The Subcommittee drafted a proposal for the BCcupm (page A-11) containing first-year core curricula for Sciences Calculus and for Social Sciences/Business Calculus courses, and implementation statements to be considered for adoption.

Background and Objectives

Background

The British Columbia Committee on the Undergraduate Programme in Mathematics (BCcupm) is the post-secondary mathematics and statistics articulation committee for the province. Its activities are conducted under the auspices of the BC Council on Admissions and Transfer (BCCAT). Articulation committees were established to foster and maintain course/program transfer arrangements to ease student movement from one educational institution to another.

At various times in the past, members of the BCcupm had advanced the thought that shaping calculus courses, particularly at some of the smaller institutions, could be significantly assisted by the existence throughout the province of greater uniformity in course content for these courses. If this were the case, not only would such institutions be helped, but students transferring from one post-secondary institution to another would also become direct beneficiaries. Thus, at its May 2001 meeting in Cranbrook, the BCcupm passed the following motion:

> **That, in order to ease transferability across all first-year calculus streams, we strike a subcommittee to develop a list of core topics for all streams of Calculus I and Calculus II.**

To implement this motion, the Meeting struck an *ad hoc* committee with representation from universities, university-colleges and colleges, and directed this subcommittee to prepare a report for the May 2002 meeting of the BCcupm. Further, to realistically support this initiative, the Meeting recommended that financial assistance be sought to ensure a reasonable round of committee activities.

Objectives

The overall goal of the Subcommittee was to accomplish the directive of the BCcupm, i.e., to develop a core curriculum for all streams of first-year calculus. To achieve this, the Subcommittee would need to attain the following objectives:

- Come to a mutual agreement concerning the primary mandate of the committee.
- Identify general issues and challenges that related to the committee's task.
- Compare the existing first-year calculus curricula for the institutions represented by members of the committee.
- Identify a set of core topics in the streams of first-year calculus.
- Develop core curricula for first-year calculus courses that would serve, in the first instance, as standards for course transfer.
- Consult during the process, where possible and as widely as prudent, with colleagues regarding the findings and the proposals of the committee.
- Prepare a report of the committee's findings and proposals for consideration by the BCcupm.

Activities

Following the impetus given this initiative, the Chair of the BCcupm proceeded immediately with the following tasks:

a) Subcommittee membership - confirmed the nominees.
b) Communication - established a means of contact among all members (electronic mail was chosen).
c) Initial Meeting - found a suitable site and a mutually convenient time.
d) Financial Support - explored the possibilities of obtaining support from BCCAT for the Core Calculus initiative and completed the application process.

Meetings of the Subcommittee were held with agendas and action plans as indicated:

Meeting (July 17, 2001, at Kwantlen University College (Richmond))

Agenda: Orientation, Clarifying Objectives/Tasks, Identifying Issues, Agreeing on Strategies, Delegating Tasks and Appointing a Chair.

Major Outcomes: Copies of current calculus course curricula for member institutions were available at the Meeting – Preliminary discussion of the Sciences Calculus with particular reference to one institution (UBC) – Bruce Kadonoff was appointed Chair – Members were to explore curricular options suggested and consult with colleagues (locally and in their constituency) prior to the next meeting.

Meeting (October 30, 2001, at Simon Fraser University (Harbour Centre))

Agenda: Project requirements, Reports of feed-back from colleagues, Identifying a Sciences Calculus Core (spreadsheet), Issues of the Social Sciences/Business Stream and Review of the objectives of this initiative.

Major Outcomes: Approval of a Transfer Innovations Project grant and its implications for this initiative were announced – Reports on preliminary reactions from colleagues as to directions proposed – Significant progress on the Sciences Core – Clarification of the project's objective: to ease calculus course transfer for students.

Meeting (January 29, 2002, at Simon Fraser University (Harbour Centre))

Agenda: Consideration of Feedback from Colleagues, Draft of Recommendations to BCcupm, Work on the Social Sciences/Business Calculus Core, Advice on the Interim Report to BCCAT.

Major Outcomes: Decisions on suggestions from colleagues regarding topics for inclusion in the Calculus Core – Preliminary decisions on proposed weighting ratios for Core/Additional Topics – Suggestion that a set of sample questions be constructed – Consideration of draft recommendations for BCcupm – Discussion of an outline of the Interim Report to BCCAT on the Transfer Innovations Project.

Meeting (March 19, 2002, at Simon Fraser University (Harbour Centre))

Agenda: Review of present status of the Core Calculus Curricula, Review of the Material Samples provided by Bruce Kadonoff, Refinement of the Recommendations for BCcupm, Discussion of the Final Report to BCCAT.

Major Outcomes: Decisions on place for the Material Samples in the Subcommittee's reports – Amplification of the recommendations statement for the BCcupm – Eliciting feedback from colleagues on the draft recommendations statement – Clarifying the character of the reports to BCcupm and to BCCAT.

Meeting (May 7, 2002, at Simon Fraser University (Harbour Centre))

Agenda: Receiving feedback on consultation with Colleagues, Final reviews of the Recommendations to BCcupm and the Core Calculus Curricula, Information and decisions regarding the completion of the Report to BCcupm.

Outcomes: Final decisions on the Recommendations to BCcupm and the Core Calculus Curricula – Directions concerning the Report to BCcupm – Discussion of the reporting session at the May Meeting of BCcupm.

In addition to their other professional duties and outside of committee meeting sessions, members of the Subcommittee devoted considerable time to such tasks as preparing draft documents, analyzing course outlines, reviewing support materials and consulting with colleagues. Each member attended every meeting and members always came to these sessions prepared to report feedback from their constituency and to move forward.

Findings

After having made informal comparisons of all the first-year calculus courses offered by each of the institutions represented by committee members, the committee focused on identifying the common topics for these courses at Simon Fraser University, The University of BC and the University of Victoria. The topics common to courses at these institutions are recorded in Appendices A and B. Further, the two streams chosen for this comparison were the Sciences Calculus and Social Sciences/Business Calculus. The Sciences Calculus courses contained more common topics than did those of Social Sciences/Business Calculus.

Besides examining the principal issue, that of the difficulties faced by mathematics departments attempting to prepare students for transfer to a number of other post-secondary educational institutions, the Subcommittee also explicitly identified some general considerations worth noting as work proceeded. Although narrowly not in its mandate, the committee realized that a mutual understanding of these related issues and challenges was critical to any expectations for success.

Some of the issues considered were:

- There is an inter-flow of students between the sciences and the non-sciences programs even within individual post-secondary institutions.
- Post-secondary institutions are autonomous.
- Prerequisites for entry to present calculus courses are not consistent throughout the province.
- In calculus course delivery, there are differences in the sequence in which topics are delivered from institution to institution.
- Practices with respect to the inclusion of technology both in instruction and in evaluation differ within institutions and among them.
- Can core curricula be so described as to avoid a gradual degeneration of some existing calculus programs? Maintaining standards is vital.

In preparing to formulate a set of core topics for first-year calculus, the committee came to the following agreements:

a) The core calculus curricula developed by the committee would be described for first-year courses. Recommendations on how to divide these into semester courses would not be made.

b) Given the almost universal practice in the BC post-secondary system of having separate calculus courses for the Sciences and for the Social Sciences/Business, the committee determined also to treat these individually.

c) The committee felt that the primary purpose for establishing core curricula in calculus was to ease the course transfer difficulties that beset institutions in developing appropriate courses and that frustrate students whose calculus backgrounds fail to meet expectations in follow-on courses.

The committee's main activity was the development of the Core Curricula that appear next in this report. However, throughout the process and between committee meetings, members took time to consult and confer with colleagues both at their own institutions and with others in their constituency. The feedback received was valuable and, although respondents would like to have expressed opinions on a final product, indicated the existence of general interest in and support for this initiative.

The diversity of perspectives among members of the committee was an important factor in achieving a balanced set of core curricula with a clear proposal.

When stepping away from the detail associated with the activities of the Subcommittee, it is noteworthy that they bear an eerie similarity to those occurring regularly in most mathematics departments when developing courses for large student populations.

Core Curriculum – Sciences Calculus

First-Year

A first year (two-semester) Sciences Calculus course must include all the topics from the **Core Topics** list. It is expected that coverage of this material would constitute three-quarters of the course(s) with the remaining one-quarter chosen from the **Additional Topics** list. For breadth, at least four Additional Topics should be included.

Reference Text: Edwards & Penney, *Calculus, Early Transcendentals, Fifth Edition*, Prentice Hall,1998.

Core Topics (75%)

1. Limits, continuity, intermediate value theorem
2. Differentiation
    - First and second derivatives with geometric and physical interpretations
    - Mean value theorem
    - Derivatives of exp and log functions, exponential growth and decay
    - Derivatives of trigonometric functions and their inverses
    - Differentiation rules (including chain rule, implicit differentiation)
    - Linear approximations and Newton's Method
    - Optimization - local and absolute extrema with applications
3. Taylor polynomials and special Taylor series (sin, cos, exp, 1/(1-x))
4. Curve sketching
5. Integration
    - Definition of the definite integral
    - Areas of plane regions
    - Average value of a function
    - Fundamental Theorem of Calculus
    - Integration techniques: substitution (including trig substitution), parts, tables, partial fractions
    - Applications of integration
6. Numerical Integration (including The Trapezoidal Rule)
7. Improper integrals: evaluation and convergence estimates
8. Differential equations (first-order linear) with applications

Additional Topics (25%)

1. Sequences and series
2. Arc length, volumes, centroids, surface areas
3. Additional differential equations topics
4. Complex numbers
5. Continuous probability density functions
6. Polar coordinates and parametric equations (with calculus applications)
7. Additional numerical methods (eg. Simpson's Rule)
8. Related rates
9. L'Hôpital's Rule

Core Curriculum
Social Sciences/Business Calculus

First-Year

A first year (two-semester) Social Sciences/Business Calculus course must include all the topics from the **Core Topics** list. It is expected that coverage of this material would constitute approximately two-thirds of the course(s) with the remaining one-third chosen from the **Additional Topics** list. For breadth, at least four Additional Topics should be included.

Reference Text: Haeussler and Paul, *Introductory Mathematical Analysis for Business, Economics, and the Life and Social Sciences, Ninth Edition*, Prentice Hall, 1998.

Core Topics (67%)

1. Limits, continuity, intermediate value theorem
2. Differentiation
    - First and second derivatives with geometrical and physical interpretations
    - Applications to economics, business and social sciences
    - Derivatives of exp and log functions, exponential growth and decay with applications
    - Derivatives of trigonometric functions
    - Differentiation rules (including chain rule, implicit differentiation)
    - Linear approximations and Newton's Method
    - Optimization - local and absolute extrema with applications
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    - Definition of the definite integral
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    - Integration techniques: substitution, parts, tables
    - Applications of integration
5. Numerical integration (including The Trapezoidal Rule)
6. Differential equations (first-order linear) with applications

Additional Topics (33%)

1. Introduction to probability and statistics
2. Partial derivatives and Lagrange multipliers
3. Matrix analysis and Gaussian Elimination
4. Sequences and series
5. Arc length, volumes, centroids, surface areas
6. Taylor polynomials and special Taylor series (sin, cos, exp, 1/(1-x))
7. Improper integrals: evaluation and convergence estimates
8. Continuous probability density functions
9. Related rates
10. Derivatives of inverse trigonometric functions
11. Further techniques of integration
12. Additional numerical integration methods

Transfer Proposal for First-Year Calculus

The BCcupm affirms the autonomy of BC's post-secondary institutions in their freedom to design calculus courses to meet the needs of their unique constituencies. However, the diversity of calculus courses in first year offerings in these institutions has created difficulties for transferring students and their institutions. For example, while current first-year "business" calculus courses at SFU, UBC and UVic share many common topics, the additional material covered by these institutions is irreconcilable within a single two-term calculus sequence.

This proposal addresses the significant challenges encountered by students transferring first-year calculus courses and by their sending institutions. To lessen the impact of these challenges, we propose that all post-secondary institutions in BC recognize a common curriculum for first-year calculus courses. Such recognition will have the following benefits for students and their institutions:

a) Provide transferring students with a solid background for subsequent math courses requiring first-year calculus.
b) Allow primarily sending institutions to design calculus courses that will meet the needs of their students post-transfer.
c) Guide primarily receiving institutions in assessing the adequacy of courses proposed for transfer.

Recommendations:

1. That the BCcupm accept the report of the First-Year Calculus Subcommittee and endorse the Core Sciences Calculus and Core Social Sciences/Business Calculus curricula as described in it.

2. That receiving institutions grant full transfer credit to first-year calculus courses from other BC post-secondary institutions whose courses are consistent with the curricula as described in this report.

3. That, when designing or modifying first-year calculus courses, all BC post-secondary mathematics departments strive to include within their courses the calculus topics as described in this report.

4. That any post-secondary institution sensing that the Core Calculus curricula as described in this report require a full or partial review raise its concerns at the next regularly scheduled meeting of the BCcupm.

5. That, in the absence of an earlier full review, the Core Calculus curricula be subject to a mandatory, full review after five years.

Appendix A - Sciences Calculus - Included Topics Comparison

Topic	Edwards & Penney	SFU Math151	SFU Math152	UBC Math100	UBC Math101	UVic Math100	UVic Math101	All
Functions and graphs	Chap 1			Yes				
Limits and Continuity	Chap 2	Yes		Yes		Yes		□
Differentiation	Chap 3	Yes		Yes		Yes		□
Additional Applications	Chap 4	Yes		Yes		Yes		□
Integration								
Antiderivatives	5.2	Yes				Yes		
The integral test	5.3		Yes		Yes	Yes		□
Riemann Sums	5.4		Yes		Yes	Yes		□
Evaluation of Integrals	5.5		Yes		Yes	Yes		□
Fundamental Theorem	5.6		Yes		Yes	Yes		□
Substitution	5.7		Yes		Yes	Yes		□
Areas of planes	5.8		Yes		Yes	Yes		□
Numerical integration	5.9		Yes		Yes	Yes		□
Applications								
Setting up integrals	6.1		Yes		Yes		Yes	□
Volumes	6.2,6.3		Yes		Yes		Yes	□
Arc length	6.4		Yes		Yes		Yes	□
Separable ODE	6.5		Yes		Yes		Yes	□
Force and Work	6.6		Yes		Yes			
Exp and Log functions	Chap 7	Yes		Yes		Yes		□
Linear 1st order ODE	7.6				Yes	Yes		
Transcendental functions								
Inverse trig functions	8.2	Yes		Yes			Yes	□
L'Hôpital's rule	8.3,8.4	Yes		Yes			Yes	□
Hyperbolic functions	8.5	Yes					Yes	
Techniques of Integration	Chap 9		Yes		Yes		Yes	□
Polar Coordinates								
Conic sections	10.1,10.2	Yes					Yes	
Area computations in polar	10.3		Yes				Yes	
Parametric curves	10.4	Yes					Yes	
Integration in parametrics	10.5		Yes				Yes	
Infinite Series								
Infinite Sequences	11.2		Yes				Yes	
Convergence	11.3		Yes				Yes	
Taylor Series	11.4		Yes	Yes			Yes	□
The integral test	11.5		Yes					
Comparison tests	11.6		Yes					
Alternating series	11.7		Yes				Yes	
Power series	11.8 11.9		Yes	Yes			Yes	□
Probability and Statistics	Handouts				Yes			
Ordinary Diff Equations								
Separable ODE	Handouts				Yes			
2nd order homogeneous	Handouts				Yes			
Non-homogeneous	Handouts				Yes			
Variation of parameters	Handouts				Yes			
Applications	Handouts				Yes			
Complex numbers	Handouts		Yes		Yes			

Appendix B - Social Sciences/Business Calculus - Included Topics Comparison

Topic	Haeussler & Paul	SFU Math157	SFU Math158	UBC Math104	UBC Math105	UVic Math102	UVic Math103	All
Non-Calculus Topics								
Functions review	Chap 3 and 4	Yes		Yes		Yes		□
Exp and log review	Chap 5	Yes		Yes		Yes		□
Matrix algebra	Chap 6		Yes				Yes	
Linear Programming	Chap 7		Yes					
Compound interest and PV	8.1 8.2			Yes		Yes	Yes (+8.3)	
Probability and Statistics	9.1 - 9.4, 10.1,10.2				Yes			
Limits and Continuity	Chap 11	Yes		Yes		Yes		□
Differentiation								
Definition	12.1,12.2	Yes		Yes		Yes		□
Rates of change	12.3	Yes		Yes		Yes		□
Continuity	12.4	Yes		Yes		Yes		□
Product and Quotient	12.5	Yes		Yes		Yes		□
Chain and Power	12.6	Yes		Yes		Yes		□
Der of log and exp	13.1,13.2	Yes		Yes		Yes		□
Implicit diff	13.3	Yes		Yes		Yes		□
Logarithmic diff	13.4	Yes						
Higher order	13.5	Yes		Yes		Yes		□
Der of trig functions	Ed & Pen 3.7			Yes		Yes		
Der of Inv trig functions	Ed & Pen 8.2			Yes				
Taylor Series	Ed & Pen 11.4			Yes				
Curve Sketching	Chap 14	Yes		Yes		Yes		□
Applications								
Applied max/min	15.1	Yes		Yes		Yes		□
Differentials	15.2	Yes		Yes		Yes		□
Elasticity of demand	15.3	Yes		Yes				
Newton's Method	15.4			Yes				
Integration								
Indefinate integration	16.1		Yes		Yes	Yes		□
Initial conditions	16.2		Yes		Yes	Yes		□
Subsitution	16.3,16.4		Yes		Yes	Yes		□
Definite integral	16.5,16.6		Yes		Yes	Yes		□
Fundamental theorem	16.7		Yes		Yes	Yes		□
Area between curves	16.8,16.9		Yes		Yes	Yes		□
Applications of area	16.10		Yes		Yes	Yes		□
Integration by parts	17.1		Yes		Yes			
Integration by partial fraction	17.2		Yes		Yes			
Integration by tables	17.3		Yes		Yes			
Average value	17.4		Yes		Yes	Yes		□
Approximation (linear)	17.5		Yes		Yes			
Simpson's Rule	17.5		Yes		Yes			
Improper Integrals	17.8		Yes		Yes			
Continuous random variables	18.1,18.2				Yes			
Differential Equations	17.6,17.7		Yes		Yes	Yes		□
Multivariable Calculus								
Partial derivatives	19.1,19.2	Yes			Yes		Yes	□
Applications	19.3	Yes			Yes		Yes	□
Implicit	19.4	Yes			Yes		Yes	□
Higher order	19.5	Yes			Yes		Yes	□
Chain Rule	19.6	Yes			Yes		Yes	□
Max / min in two variables	19.7	Yes			Yes		Yes	□
Lagrange multipliers	19.8	Yes			Yes		Yes	□
Lines of regression	19.9	Yes						

Appendix C - The Core Calculus Subcommittee

The Core Calculus Subcommittee was struck by the BCcupm at its May 2001 meeting in Cranbrook, British Columbia, as an *ad hoc* committee to "develop a set of core topics" for first-year calculus courses. The committee's membership was to include representation as follows: one each from Simon Fraser University, The University of BC, the University of Victoria, a university college and a college. The Chair of the BCcupm was to act as a facilitator and as an *ex officio* member of the committee.

The First-Year Core Calculus Subcommittee members were:

Name	Institution	Transfer Perspective
Dr. Rustum Choksi	Simon Fraser University	Receiving
Bruce Kadonoff (Chair)	Coquitlam College	Sending
Dr. David Leeming	University of Victoria	Receiving
Dr. Philip Loewen	The University of BC	Receiving
Dr. Casey McConill	Kwantlen University College	Sending
Leo Neufeld (Project Contractor)	BCcupm	Sending